Algebra II — Summary 5 - Trigonometry

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The *radian measure* of an angle is the arclength of the arc on a circle of radius 1 which subtends the angle when the vertex is placed at the center of the circle. We can convert between degrees and radians by using the fact that

$$360^\circ = 2\pi \text{ radians.}$$

The *wrapping function* $P : \mathbb{R} \to \mathbb{R}^2$ is defined to be the point $P(\theta)$ on the unit circle $x^2 + y^2 = 1$ obtained by moving counterclockwise along the circle an arclength of θ. If $\theta < 0$, this is interpreted as moving clockwise by an arclength of $|\theta|$.

The *sine* and *cosine* functions as defined by

$$\sin\theta = \text{ the } y\text{-coordinate of } P(\theta) \quad \text{and} \quad \cos\theta = \text{ the } x\text{-coordinate of } P(\theta).$$

It is clear that

$$\sin^2\theta + \cos^2\theta = 1.$$

The *tangent*, *cotangent*, *secant*, and *cosecant* are defined by

$$\tan\theta = \frac{\sin\theta}{\cos\theta}, \quad \cot\theta = \frac{\cos\theta}{\sin\theta}, \quad \sec\theta = \frac{1}{\cos\theta}, \quad \csc\theta = \frac{1}{\sin\theta}.$$

Trigonometric Identities

Identities that Come from Geometry

Description	Identity
Pythagorean Identity	$\cos^2\theta + \sin^2\theta = 1$
Symmetry	$\sin(-\theta) = -\sin\theta$ $\cos(-\theta) = \cos\theta$
Periodicity	$\sin(\theta + 2\pi) = \sin\theta$ $\cos(\theta + 2\pi) = \cos\theta$
Rotation by 180°	$\sin(\theta + \pi) = -\sin\theta$ $\cos(\theta + \pi) = -\cos\theta$
Rotation by 90°	$\sin(\theta + \frac{\pi}{2}) = \cos\theta$ $\cos(\theta - \frac{\pi}{2}) = \sin\theta$
Reflection across 45°	$\sin(\frac{\pi}{2} - \theta) = \cos\theta$ $\cos(\frac{\pi}{2} - \theta) = \sin\theta$
Difference of Angles Formula	$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

Identities that Come from Algebra

Description	Identity
Sum and Difference Formulas	$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$ $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha$
Double Angle Formulas	$\cos(2\theta) = \cos^2\theta - \sin^2\theta$ $\sin(2\theta) = 2\sin\theta\cos\theta$
Half Angle Formulas	$\sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}}$ $\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}}$
Pythagorean Formulae	$1 + \tan^2\theta = \sec^2\theta$ $\cot^2\theta + 1 = \csc^2\theta$

Trigonometric Values

deg(θ)	rad(θ)	sin(θ)	cos(θ)	tan(θ)	cot(θ)	sec(θ)	csc(θ)
0°	0	0	1	0	∞	1	∞
15°	$\frac{\pi}{12}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{2-\sqrt{3}}{2}$	$\frac{2+\sqrt{3}}{2}$	$\sqrt{6}-\sqrt{2}$	$\sqrt{6}+\sqrt{2}$
18°	$\frac{\pi}{10}$	$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{5-2\sqrt{5}}{2}$	$\sqrt{5+2\sqrt{5}}$	$\sqrt{2\sqrt{5}-5}$	$\sqrt{5}+1$
	$\frac{\pi}{8}$	$\frac{\sqrt{2-\sqrt{2}}}{2}$	$\frac{\sqrt{2+\sqrt{2}}}{2}$	$\sqrt{3-2\sqrt{2}}$	$\sqrt{3+2\sqrt{2}}$	$\sqrt{4-2\sqrt{2}}$	$\sqrt{4+2\sqrt{2}}$
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2
36°	$\frac{\pi}{5}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{1+\sqrt{5}}{4}$	$\sqrt{5-2\sqrt{5}}$	$\frac{\sqrt{25+10\sqrt{5}}}{5}$	$\sqrt{5}-1$	$\frac{10+2\sqrt{5}}{5}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
54°	$\frac{3\pi}{10}$	$\frac{1+\sqrt{5}}{4}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{25+10\sqrt{5}}}{5}$	$\sqrt{5-2\sqrt{5}}$	$\frac{10+2\sqrt{5}}{5}$	$\sqrt{5}-1$
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$
	$\frac{3\pi}{8}$	$\frac{\sqrt{2+\sqrt{2}}}{2}$	$\frac{\sqrt{2-\sqrt{2}}}{2}$	$\sqrt{3+2\sqrt{2}}$	$\sqrt{3-2\sqrt{2}}$	$\sqrt{4+2\sqrt{2}}$	$\sqrt{4-2\sqrt{2}}$
72°	$\frac{2\pi}{5}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{5}-1}{4}$	$\sqrt{5+2\sqrt{5}}$	$\frac{5-2\sqrt{5}}{2}$	$\sqrt{5}+1$	$\sqrt{2\sqrt{5}-5}$
75°	$\frac{5\pi}{12}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{2+\sqrt{3}}{2}$	$\frac{2-\sqrt{3}}{2}$	$\sqrt{6}+\sqrt{2}$	$\sqrt{6}-\sqrt{2}$
90°	$\frac{\pi}{2}$	1	0	∞	0	∞	1